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1989 J. Phys.: Condens. Matter 1 1025

(http://iopscience.iop.org/0953-8984/1/5/021)

J. Phys.: Condens. Matter **1** (1989) 1025–1027. Printed in the UK

LETTER TO THE EDITOR

Quantised thermopower of a channel in the ballistic regime

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Received 22 November 1988

Abstract. It is shown that the quantised nature of the ballistic conductance in a narrow channel leads to the peak structure of the thermopower with quantised values of local maxima. The correspondence with the properties of the two-dimensional electron gas in quantising magnetic fields is found.

Recent experimental studies (van Wees *et al* 1988, Wharam *et al* 1988) have shown that the conductance of a short narrow channel acquires quantised values $2ie^2/h$, where i is the number of the occupied sub-bands. The twofold spin degeneracy of each sub-band is considered. The result may be readily derived from the Landauer approach (Landauer 1987, Büttiker 1986) which relates the conductance to the transmission coefficient. In the absence of scattering it essentially states that, due to the exact compensation between the density of states at the Fermi energy and the Fermi velocity, each sub-band contributes the same current. By altering the number of occupied sub-bands by changing the channel width or the carrier concentration, the conductance is changed from one quantised value to another. Such a step-like dependence can be observed at low temperatures when the transport through a channel may be regarded as truly ballistic (van Wees *et al* 1988, Wharam *et al* 1988). The studies are usually performed on the so-called Sharvin contacts (Sharvin 1965) realised as a constriction within a two-dimensional electron system. The measured conductance or resistance may be considered as the two-terminal resistance of a device composed of two particle reservoirs connected together by a narrow channel. In contrast to the case for the conductance, little attention has so far been paid to the thermopower of such systems.

To establish the channel contribution to the static thermopower, let us study the following model system. Each of the reservoirs is supposed to have such a high degree of thermal and electrical conductivity that it is able to keep the electron system in thermal equilibrium independently of the transport properties of the narrow channel connecting them. Furthermore, an ideal channel uniform along the current flow is supposed.

Within the linear response approach the total electric current J through the channel is proportional to the difference of the chemical potentials $\Delta\mu$ between the reservoirs and to their temperature difference ΔT. With the help of the energy-dependent transmission

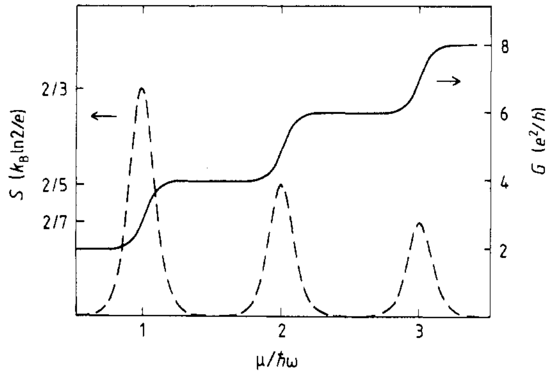

Figure 1. The two-terminal conductance G (full curve) and the thermopower S (broken curve) of a channel in the form of the parabolic potential well as functions of the chemical potential $\mu/\hbar\omega$ ($\hbar\omega$ denotes the sub-band spacing, $k_BT/\hbar\omega = 0.05$).

coefficient $t(E)$ we have (Sivan and Imry 1986)

$$J = -2\frac{e^2}{h}\int_{-\infty}^{+\infty}\frac{\mathrm{d}n_F(E)}{\mathrm{d}E}t(E)\left(\frac{E-\mu}{T}\Delta T + \Delta\mu\right)\mathrm{d}E \tag{1}$$

where $n_F(E)$ stands for the Fermi–Dirac distribution function, μ and T denote the average chemical potential and temperature, respectively. Keeping in mind the condition $J = 0$ we readily obtain the Cutler–Mott expression for the static thermopower S (Cutler and Mott 1969):

$$S = -\frac{\Delta\mu}{e\Delta T} = \frac{k_B}{e}\int_{-\infty}^{+\infty}\frac{\mathrm{d}n_F(E)}{\mathrm{d}E}t(E)\frac{E-\mu}{k_BT}\mathrm{d}E\left(\int_{-\infty}^{+\infty}\frac{\mathrm{d}n_F(E)}{\mathrm{d}E}t(E)\,\mathrm{d}E\right)^{-1} \tag{2}$$

where $-e$ is the absolute value of the electron charge and $2e^2t(E)/h$ represents the two-terminal conductance (Landauer 1987, Büttiker 1986) at zero temperature.

In the ballistic transport regime the transmission coefficient of an ideal narrow channel may be approximated by a step function of energy and we get

$$S = \frac{k_B}{e}\left[\sum_{n=0}^{+\infty}\int_{E_n}^{+\infty}\left(-\frac{\mathrm{d}n_F(E)}{\mathrm{d}E}\right)\frac{E-\mu}{k_BT}\mathrm{d}E\right]\left(\sum_{n=0}^{+\infty}n_F(E_n)\right)^{-1} \tag{3}$$

where E_n denotes the lowest energy of the nth sub-band. At low temperatures when the thermal energy k_BT is much less than the sub-band spacing we found that the thermopower differs from zero only when the reduced chemical potential $\mu - E_n$ lies within an energy interval the width of which is approximately k_BT. When μ equals the lowest energy of the ith sub-band the thermopower approaches a local maximum. Its height is temperature independent and acquires the quantised value

$$S_i^{\max} = \frac{k_B}{e}\frac{\ln 2}{i+\frac{1}{2}} \simeq -\frac{60}{i+\frac{1}{2}} \quad (\mu\mathrm{V\,K}^{-1}). \tag{4}$$

The chemical potential dependence of a finite-temperature conductance and thermopower for the two-dimensional channel described by a parabolic potential well is plotted in figure 1.

In the case of a non-ideal channel which is non-uniform along the direction of current flow the transmission coefficient is changed by unity within a finite energy interval Γ. When the condition $k_B T \gg \Gamma$ is satisfied the properties derived for an idealised channel remain valid and the thermopower is a universal function of the reduced chemical potential and the sub-band spacing.

In the ballistic transport regime the contribution of a narrow channel to the static thermopower, (3) and (4), is just equal to the diffusion part of the diagonal component of the thermopower S_{xx} of a two-dimensional electron system in the quantising magnetic field (Girvin and Jonson 1982, Streda 1983). The thermopower as well as the two-terminal conductance (Streda *et al* 1987) is thus independent of the origin of the energy structure quantisation. Both the magnetic and size quantisation yield formally the same results which do not obey Mott's rule.

It is well known that at low temperatures the phonon drag contribution to the thermopower may prevail (Fletcher *et al* 1988, Ruf *et al* 1988). Since in the ballistic transport regime the electron–phonon interaction vanishes within the channel, the deviations from the properties described can be caused only by the properties of the reservoirs. Due to the phonon drag effect the distribution of electrons fed into the channel may not be the equilibrium one in the presence of a thermal gradient through the real devices.

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